Black hole instabilities and violation of the weak cosmic censorship in higher dimensions

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work in progress

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Why gravity in higher $D$?

- Study fundamental aspects of gravity in new settings
- String theory, AdS/CFT
- GR simplifies in the large $D$ limit
- New gravitational physics in $D>4$:
  1. Gravitational instabilities [Gregory and Laflamme]
  2. New black hole topologies [Emparan and Reall; Schoen and Galloway]
Outline of the talk

• Motivation: the weak cosmic censorship conjecture
• Black ring instabilities
• Rotating spherical black hole instabilities
• Summary and conclusions
The weak cosmic censorship conjecture

- GR has a well-posed initial value problem [Choquet-Bruhat; Choquet-Bruhat and Geroch; Sbierski]

- Singularity theorems in GR: singularities form generically [Penrose; Hawking and Penrose]

- If singularities form generically, does GR have any predictive power at all?

- What kind of singularities form generically in dynamical evolution?
The weak cosmic censorship conjecture

“Generic asymptotically flat initial data have a maximal future development possessing a complete future null infinity”

[Penrose; Geroch and Horowitz; Christodoulou]

• If a black hole is unstable, can the singularity inside become visible during the evolution?
The Gregory-Laflamme instability for black strings

- Black strings: black hole solution of the Einstein vacuum equations in $M_4 \times S^1$

$$ds^2 = - \left( 1 - \frac{2M}{r} \right) dt^2 + \frac{dr^2}{1 - \frac{2M}{r}} + r^2 d\Omega_{(2)}^2 + dz^2 \quad \hspace{1cm} z \sim z + L$$

- If $M/L \lesssim O(1)$ black strings are unstable to develop ripples along the compact extra dimension [Gregory and Laflamme]
• The weak cosmic censorship conjecture may be false in spaces with compact extra dimensions

• No fine-tuning is required

• The horizon develops a fractal structure

• Self-similar process

• The black string breaks in finite asymptotic time

• The weak cosmic censorship conjecture may be false in spaces with compact extra dimensions

[Lehner and Pretorius]
Can the weak cosmic censorship conjecture be violated around higher dimensional asymptotically flat black hole spacetimes?
Black ring instabilities
Black hole phases in 5D

\( M = 1 \)
Black hole phases in 5D

- Non-uniqueness
- Unstable radially

unstable to GL [Santos&Way]

[Elvang,Emparan&Virmani; PF,Murata&Reall]
What is the endpoint of the instabilities?

Does weak cosmic censorship hold around black ring spacetimes?
Black strings

Spherical black holes
• However the computations were very expensive (it’s a 3+1 problem) and the understanding of the endpoint was limited:

  - Time-scale of the pinch-off could not be estimated

  - Is the process self-similar as in black strings?

Can we understand the details of the Gregory-Laflamme instability in asymptotically flat spaces?
Rotating spherical black hole instabilities
Myers-Perry BHs in $D \geq 6$

- The higher dimensional analogues of the Kerr BH:

$$ds^2 = -dt^2 + \frac{\mu}{r} \sum (dt - a \sin^2 \theta d\phi)^2 + \frac{\Sigma}{\Delta} dr^2 + \Sigma d\theta^2$$

$$+ (r^2 + a^2) \sin^2 \theta d\phi^2 + r^2 \cos^2 \theta d\Omega^2_{(D-4)}$$

$$\Sigma = r^2 + a^2 \cos^2 \theta \quad \Delta = r^2 + a^2 - \frac{\mu}{r^{D-5}}$$ [Myers and Perry]

- In $D \geq 6$ MP black holes can rotate arbitrarily fast

- In the limit $a \to \infty$, MP black holes resemble black membranes, which are unstable under the Gregory-Laflamme instability [Emparan and Myers]
Black hole phases in $D \geq 6$

[Emparan and Myers; Emparan et al., PF et al., Dias et al., ...]
\[ \frac{t}{\mu^3} = 30.0000 \]
10,000 thinner than the original black hole!!!
The local geometry is well approximated by a sequence of black rings connected by black membranes.

The outermost ring carries most of the mass and angular momentum.
• Differences between the dynamics of black strings and ultra spinning MP black holes:
  - Boundary effects are important initially
  - Centrifugal force: non-uniform membrane sections
  - Motion of higher generation rings
The evolution of the ultra spinning instability of MP black holes is NOT self-similar
Evolution

- The minimum thickness follows a scaling law: $Z_{AH} = \alpha(t_c - t)$
Summary and Conclusions
Summary and Conclusions

- Black rings and ultraspinning MP black holes are unstable and the instability evolves into a naked singularity in finite asymptotic time.

- The weak cosmic censorship conjecture around ultraspinning MP black holes and black rings may be false.

- This is generic in higher dimensions.
• Evolution of non-axisymmetric instabilities of spherical black holes
• **Conjecture 1**

The Gregory-Laflamme instability is the only mechanism that GR has to change the horizon topology

• **Conjecture 2**

The only stable black hole in $D>4$ is the Myers-Perry solution with $J/M^{D-3}=O(1)$
Thank you for your attention!