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VOLKER BACH, Institut für Analysis und Algebra TU Braunschweig
The Dynamical Renormalization Group - Beyond the van Hove Time Scale

Given an arbitrarily large, but fixed, time $t > 0$, we derive approximations for the time evolution of the spin-boson model in terms of the propagator generated by a free effective Hamiltonian. For initial states spectrally localized close to a resonance energy, the error made by these approximations tends to zero, as t grows large, even compared to the exponential decay induced by the resonance. Our construction rests on the renormalization group induced by the isospectral Feshbach-Schur map. This is joint work with Jacob Schach Möller and Matthias Westrich.

ALAIN JOYE, Université Grenoble Alpes, Institut Fourier
Chirality induced Interface Currents in the Chalker Coddington Model

Chalker $\&$ Coddington provided in 1988 a simplified description of the quantum dynamics of electrons in a plane, submitted to an electric potential and a strong perpendicular magnetic field, in a model that now bears their names. The one time step electronic motion is given by a unitary operator on $l^2(\mathbb{Z}^2)$ constructed in terms of scattering matrices attached to the sites of $\mathbb{Z}^2$ that contain the main physical characteristics of the potential and magnetic field at these sites. The transport properties of the electrons are then encoded in the spectral properties of the unitary operator, which is our main concern. We consider the situation where the model presents asymptotically pure anti-clockwise rotation on the left and clockwise rotation on the right and we investigate the presence of induced currents at the interface between these two different localised phases. The existence of interface currents is shown by proving that the absolutely continuous spectrum of the Chalker Coddington unitary operator covers the whole unit circle. The result is independent of the details of the model within the interface and possesses some topological features. This is joint work with J.Asch and O.Bourget

DINH–THI NGUYEN, Mathematisches Institut der Universität München
Many-body blow-up of boson stars

We study ground states of a system of N identical bosons in $\mathbb{R}^3$, described by the Hamiltonian

$$H_N = \sum_{i=1}^{N} \left(\sqrt{-\Delta_{x_i} + m^2} + V(x_i) \right) - \frac{a}{N-1} \sum_{1 \le i < j \le N} |x_i - x_j|^{-1}.$$

acting on Hilbert space $\bigotimes_{\mathrm{sym}}^{N} L^2(\mathbb{R}^3)$. Here the parameter $m > 0$ is the mass of particles, $a > 0$ describes the strength of the attractive interaction, and $V \geq 0$ is an external potential. We are interested in the behavior of the ground state energy per particle of H_N and the corresponding ground state when $N \to \infty$ and $a = a_N$ tends to a^* *(Chandrasekhar limit)* from below. We first study blow-up behavior of ground state energy as well as of ground states when $a \nearrow a^*$ in the effective model: *Hartree theory*.

JEAN NOURRIGAT, University of Reims
Infinite dimensional semiclassical analysis and applications to a model in NMR.

We are interested in the approximation of the quantum evolution of observables for N fixed particles with spin 1/2, interacting with the quantized electromagnetic field, and a constant magnetic field. This is the Nuclear Magnetic Resonance, modelized in the framework of Quantum Electrodynamics, closely to the spin-boson model.

The observables may be the electric or magnetic field at each point, or a spin observable. If A is such an operator, and if H is the Hamiltonian, we study the Wick symbol of $e^{i(t/h)H}Ae^{-i(t/h)H}$. Formally, it has a semiclassical expansion, each term satisfying a differential equation given by the Mizrahi series (which gives the Wick symbol of a composed operator).

The first term satisfies the Bloch equations (1946), coupled with the Maxwell system. All the others give quantum corrections to these equations.

In order to control the error, the main point is that, for each function F on the (infinite dimensional) phase space satisfying suitable estimations for its differentials of all orders, we can find an operator whose Wick symbol is F, and we can control its norm. Similar techniques are used in the pseudodifferential calculus in $\mathbb{R}^n$ and, in the Fock space, for polynomials (Wick order). All our terms satisfy these conditions, and the Wick quantization enables us to estimate the error for the Wick symbol.

(With L. Amour and L. Jager, arXiv:1705.07097, new version march 2018.)

MARIA CLARA NUCCI, University of Perugia and INFN-Perugia, Italy
Hidden Noether symmetries in quantum mechanics

In 1918 Noether published her landmark paper. In the past one hundred years her namesake theorem has been applied in different area of Physics, especially general relativity and classical Lagrangian mechanics. This talk aims to celebrate this landmark centennial by showing the application of Noether symmetries in the quantization of classical mechanics, a quantization method that preserves the Noether point symmetries as we have described for the first time in 2011, and since then successfully applied to various classical problems, e.g. families of Lienard nonlinear oscillators, N planar rotors, a radial harmonic oscillator on a double cone.

SOEREN PETRAT, Jacobs University
Derivation of the Bogoliubov Time Evolution for Bose Gases with Finite Speed of Sound

We consider the dynamics of an interacting Bose gas near the ground state in a limit where both the volume and the density of the gas tend to infinity. The interaction is scaled with the inverse density. We prove a mean-field type result and consider the Bogoliubov excitations around a product state. This allows us to prove convergence of the N-body dynamics to the Bogoliubov approximation in L^2 norm, while the Hartree approximation gives convergence only in the sense of reduced densities. These results can be applied to the setting of a Bose gas with slight perturbations. Then the coupling constant is such that the self-interaction of the fluctuations is of leading order, which leads to a finite (non-zero) speed of sound in the gas.

ANDREA SACCHETTI, Università di Modena e Reggio Emilia
Bifurcation trees in nonlinear Schrödinger equations

In this talk we discuss some recent results for a class of nonlinear models in Quantum Mechanics. In particular we focus our attention to the nonlinear one-dimensional Schrödinger equation with a periodic potential and a Stark- type perturbation. In the limit of large periodic potential the Stark-Wannier ladders of the linear equation become a dense energy spectrum because a cascade of bifurcations of stationary solutions occurs when the ratio between the effective nonlinearity strength and the tilt of the external field increases.

ITARU SASAKI, Shinshu University
Existence of ground state of massless relativistic Pauli-Fierz model

We consider the massless relativistic Pauli-Fierz model which describes the massless charged particle interacting with a quantized radiation field. The Hamiltonian of this model have the form $H = |p - eA(x)| + H_f + V(x)$ where p is the momentum of the particle, $A(x)$ is the quantized vector potential, H_f is the free energy of the photon and $V(x)$ is the external potential. We show the existence of ground state of this model under suitable condition for V.

JÉRÉMY SOK, University of Basel
Dirac operators with magnetic links

The existence of zero modes for Dirac operators with magnetic fields is the cause of break down of stability of matter for charged systems.

However the known examples are geometrically complex, and a complete classification of zero modes is unknown. In particular, one does not know the characteristics of the magnetic fields which produce the zero modes.

To better understand them, we studied the particular case of magnetic fields with finitely many field lines which form a link. These singular fields can be seen as generalizations of the Aharonov-Bohm solenoids, and they exhibit the same 2π-periodicity of the fluxes carried by their field lines.

Tuning one flux from 0 to 2π gives rise to a loop of Dirac operators for which we can study the spectral flow, a non-trivial spectral flow indicating the occurence of zero modes. It turns out that this number depends on the geometry of the magnetic fields: the interlinking of the field lines but also their shapes.

(Joint work with Fabian Portmann and Jan Philip Solovej)

CARLOS VILLEGAS-BLAS, Universidad Nacional Autonoma de Mexico, Instituto de Matemáticas.
On semiclassical eigenvalue distribution theorems for perturbations of the Landau Problem

Let $H_0 = (-\imath\frac{\partial}{\partial x} + \frac{B}{2}y)^2 + (-\imath\frac{\partial}{\partial y} - \frac{B}{2}x)^2$ be the Landau Hamiltonian with spectrum given by the infinitely degenerate discrete Landau levels $\lambda_n = B(2n+1)$, with B the strength of the constant magnetic field and n a non-negative integer number. We consider perturbations given by smooth short range electric potentials V. We study two semiclassical limits of the eigenvalue distribution inside the clusters around the Landau levels generated by the perturbation. In the first one, we study the high energy limit $n \to \infty$ of the clusters associated to the operator $H = H_0 + V$ around λ_n. In the second one, we introduce the Planck parameter $\hbar$ achieving only discrete values $\hbar = E/(2n+1)$ and actually consider the unperturbed operator $\tilde{H}_0 = (-\imath\hbar\frac{\partial}{\partial x} + y)^2 + (-\imath\hbar\frac{\partial}{\partial y} - x)^2$ taking $B = 2/\hbar$. Considering the perturbation $\hbar^2 V$, we study the limit $n \to \infty$ fixing the classical energy E and looking at the eigenvalue clusters of $\tilde{H} = \tilde{H}_0 + \hbar^2 V$ around E. We obtain averages of V along straight lines on the plane and along circles on the plane with arbitrary center and fixed radius (the classical orbits with energy E) in the first and second limits respectively. This is joint work with A. Pushnitski, G Raikov, G. Hernandez Dueñas, S. Perez-Esteva and A. Uribe.

AMANDA YOUNG, University of Arizona
On the stability of frustration-free gapped ground states of quantum lattice systems

Gapped quantum spin systems with topologically ordered ground states have been of interest due to their potential for developing fault-tolerant quantum codes. A key feature of such models is that the spectral gap remains open in the presence of small, local perturbations. Several results in this direction are due to Bravyi-Hastings-Michalakis (2010), Bravyi-Hastings (2011), and Michalakis-Zwolak (2013). These results, however, are only proved for quantum spin models with periodic boundary conditions and a unique infinite volume frustration-free ground state. Our work extends these results in several directions: including to quantum spin systems with more general boundary conditions, quantum spin systems with discrete symmetry breaking, and lattice fermion models.