We consider a quantum mechanical system, which is modeled by a Hamiltonian acting on a finite dimensional space with degenerate eigenvalues coupled to a field of relativistic bosons. Provided a mild infrared assumption holds, we prove that the ground-state eigenvalue and the corresponding eigenprojection are analytic functions of the coupling constant in a cone with apex at the origin. In order to show this we extend operator theoretic renormalization to degenerate situations.