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Derivation of the 1d Gross-Pitaevskii equation from the 3d quantum many-body dynamics of strongly confined bosons

We consider the dynamics of N interacting bosons initially exhibiting Bose-Einstein condensation. Due to an external trapping potential, the bosons are strongly confined in two spatial directions, with the transverse extension of the trap being of order ε. The non-negative interaction potential is scaled such that its scattering length is positive and of order $(N/\varepsilon^2)^{-1}$ and the range of the interaction scales as $(N/\varepsilon^2)^{-\beta}$ for $\beta \in (0,1]$. We prove that in the simultaneous limit $N \to \infty$ and $\varepsilon \to 0$, the condensation is preserved by the dynamics and the time evolution is asymptotically described by a nonlinear Schrödinger equation in one dimension. The strength of the nonlinearity depends on the interaction and on the shape of the confining potential. For $\beta = 1$, the effective equation is a physically relevant one-dimensional Gross-Pitaevskii equation, where the coupling parameter contains the scattering length of the unscaled interaction. For our analysis, we adapt an approach by Pickl to the problem with dimensional reduction.