The fundamental relation between quantum entanglement and convex algebraic geometry has unveiled a set of powerful tools, imported from the former to the study the latter. The space of separable mixed states is convex and so is the space of the corresponding observable values. Therefore, the problem of determining whether a set of observable values come from an entangled state is tantamount to checking for membership in a convex set, of a point with coordinates given by the set of observable values.

Here, we use techniques from convex algebraic geometry to develop powerful criteria for entanglement in a many-body system of Bosonic atoms with a non-zero spin. The experimentally accessible observables are the spin expectation values $\langle S_i \rangle$ and $\langle \{S_i, S_j\} \rangle$ which, upon truncating at rank two, are 9 independent quantities in general. Recently, entanglement criteria in terms of 2 of these 9 quantities have been derived [1]. We develop entanglement criteria using all of these 9 quantities. If we consider these numbers as coordinates of a point in a 9 dimensional space, those with a separable parent state lie within a convex region, also known as the moment cone. Therefore, the problem is to characterize this moment cone. Owing to the bosonic exchange symmetry, this moment cone is the dual of the cone of non-negative symmetric polynomials. Together with a characterization of this cone, we also develop an entanglement criterion that is asymptotically tight, for large atom numbers.