In this talk I will compare and contrast (deterministic) chaotic dynamical systems and their stochastic counterparts, i.e. when small random perturbations are added to such systems to model uncontrolled fluctuations. Three groups of results, some old and some new, will be discussed. The first has to do with how deterministic systems, when sufficiently chaotic, produce observations resembling those from genuinely random processes. The second compares the ergodic theories of chaotic systems and of random maps (as in stochastic flows of diffeomorphisms generated by SDEs). One will see that results on SRB measures, Lyapunov exponents, entropy, fractal dimension, etc. are all nicer in the random setting. I will finish by suggesting that to improve the applicability of existing theory of chaotic systems, a little bit of random noise can go a long way.