The Conditional Entropy Power Inequality for Bosonic Quantum Systems

We prove the conditional Entropy Power Inequality for Gaussian quantum systems. This fundamental inequality determines the minimum quantum conditional von Neumann entropy of the output of the beam-splitter or of the squeezing among all the input states where the two inputs are conditionally independent given the memory and have given quantum conditional entropies. We also prove that, for any couple of values of the quantum conditional entropies of the two inputs, the minimum of the quantum conditional entropy of the output given by the conditional Entropy Power Inequality is asymptotically achieved by a suitable sequence of quantum Gaussian input states. Our proof of the conditional Entropy Power Inequality is based on a new Stam inequality for the quantum conditional Fisher information and on the determination of the universal asymptotic behaviour of the quantum conditional entropy under the heat semigroup evolution. The beam-splitter and the squeezing are the central elements of quantum optics, and can model the attenuation, the amplification and the noise of electromagnetic signals. This conditional Entropy Power Inequality will have a strong impact in quantum information and quantum cryptography. Among its many possible applications there is the proof of a new uncertainty relation for the conditional Wehrl entropy.

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