Locality at the boundary implies gap in the bulk for 2D PEPS

Proving that the parent Hamiltonian of a Projected Entangled Pair State (PEPS) is gapped remains an important open problem. We take a step forward in solving this problem by showing two results: first, we identify a condition on the boundary state of rectangular subregions that is sufficient to prove that the parent Hamiltonian of the bulk 2D PEPS has a constant gap in the thermodynamic limit; second, we then show that boundary states which are Gibbs state of a local Hamiltonian on the virtual indices with interactions decaying faster than exponentially in the distance satisfy such condition. The proof employs the martingale method of nearly commuting projectors, and exploits a result of Araki on the robustness of one dimensional Gibbs states. Our result provides one of the first rigorous connections between boundary theories and dynamical properties in an interacting many body system.