In the first part of this talk I will present work joint with Gukov and Pei, where we have derived a Verlinde formula for the space of states in complex Quantum Chern Simons theory on surfaces with labeled marked points for general simple Lie groups. Although this state space is infinite dimensional, it has a natural grading in terms of finite dimensional spaces and we give an explicit formula for the generating function for the dimensions of these. In fact we establish that these generating functions gives a one parameter family of 1-1 dimensional TQFT, which completely determines all these dimensions.

In the second part I will present joint work with Kashaev, where we have given a combinatorial explicit construction of complex Quantum Chern-Simons theory much like Reshetikhin and Turaev gave a combinatorial construction of Quantum Chern-Simons theory in the case of compact groups. We consider the non-compact groups $SL(2, \mathbb{R})$ and $SL(2, \mathbb{C})$ and we use triangulations and charged quantum Teichmüller theory to construct two versions of this TQFT. We verify that the resulting state integrals are indeed convergent and an invariant of the triangulation and thus gives well-defined invariants independent of the triangulation. I shall state our version of the volume conjecture for these invariants for the group $SL(2, \mathbb{R})$, which propose that these invariants decay exponentially in Planck’s constant at the rate given by the hyperbolic volume. I will present our proof of this conjecture for a few simple knots.