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On the canoncial commutation relations for the wave operator on static Lorentzian manifolds with timelike boundary

We consider the wave operator on static, Lorentzian manifolds with timelike boundary. The quantization of the underlying field theory is based on the identification of the commutation relations in terms of advanced and retarded fundamental solutions. Since the manifold has a non empty boundary their existence is not guaranteed a priori and it must be discussed in terms of boundary conditions. By means of spectral calculus we prove that answering this question is equivalent to studying the self-adjoint extensions of an associated elliptic operator on a Riemannian manifold with boundary (M, g), assumed to be of bounded geometry. This problem can be tackled within the framework of boundary triples. These consist of the assignment of two surjective, trace operators from the domain of the adjoint of the elliptic operator into an auxiliary Hilbert space h, which is the third datum of the triple. Self-adjoint extensions of the underlying elliptic operator are in one-to-one correspondence with self-adjoint operators Θ on h. On the one hand, we show that, for a natural choice of boundary triple, each Θ can be interpreted as the assignment of a boundary condition for the original wave operator. On the other hand, we prove that, for each such Θ, there exists a unique advanced and retarded fundamental solution. In addition, we prove that these share the same structural property of the counterparts associated to the wave operator on a globally hyperbolic spacetime. This is joint work with Nicolò Drago and Hugo Ferreira, based on 1804.03434 [math-ph]